1. Three semicircles have diameters that are the sides in a right-angled triangle. Their areas are  $X \text{ cm}^2$ ,  $Y \text{ cm}^2$  and  $Z \text{ cm}^2$ , as shown in the figure. Which of the following expressions is necessarily correct?

(1) X + Y < Z(2) $\sqrt{X} + \sqrt{Y} = \sqrt{Z}$ (3) X + Y = Z(4)  $X^2 + Y^2 = Z^2$ (5) $X^2 + Y^2 = Z$ 



- 2. There are four distinct points in space A, B, C, D, and the dot product  $\overrightarrow{AB} \cdot \overrightarrow{AC} = \overrightarrow{AB} \cdot \overrightarrow{AD}$ . Try to choose the correct option.
  - (1)  $\overrightarrow{AB} \cdot \overrightarrow{CD} = 0$
  - (2)  $\overline{AC} = \overline{AD}$
  - (3)  $\overrightarrow{AB}$  is parallel to  $\overrightarrow{CD}$
  - (4)  $\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$
  - (5) Points A, B, C, and D are on the same plane
- 3. What is the result from  $\sqrt{(2025 + 2025) + (2025 2025) + (2025 \cdot 2025) + 2025/2025}$ ?
  - (1)  $\sqrt{2025}$
  - (2) 2025
  - (3) 2026
  - (4) 2027
  - (5) 4050
- 4. Albert was calculating the volume of a sphere, but in his calculation he used, mistakenly, the value of the diameter instead of the radius of the sphere. What should he do to his wrong answer in order to obtain the correct answer?
  - (1) Divide it by 2.
  - (2) Divide it by 4.
  - (3) Divide it by 6.
  - (4) Divide it by 8.
  - (5) Divide it by 16.

5. Determine the value of the expression  $\log(29) - \log(7) + \log(12/71) - \log(348/497)$ 

- (1) -2
- (2) -1
- (3) 0
- (4) 1
- (5) 2

6. Solve the system of equations 17x + 3y = -73 15x - 13y = 139

- (1)x = -2, y = -13(2)x = -2, y = -11(3)x = 4, y = 11(4)x = 4, y = 13(5)x = 2, y = -11
- 7. Consider the polynomial  $f(x) = 3x^4 + 11x^2 4$ . Choose the correct option:
  - (1) The graph of y = f(x) intersects the y-axis at a y-coordinate larger than 0.
  - (2) f(x) = 0 has 4 real roots.
  - (3) f(x) = 0 has at least one rational root.
  - (4) f(x) = 0 has a root between 0 and 1.
  - (5) f(x) = 0 has a root between 1 and 2.
- 8. An online seller acquires a model at a cost of 200 dollars and sets the selling price as five times the cost, with the difference being the profit. However, after some time with no inquiries, the seller decides to reduce the selling price by halving the profit each time. Following this approach, after three reductions, how much is the selling price in dollars?

(1) 400 (2) 300 (3) 250 (4) 225 (5) 212.5

9. Assume  $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ . If  $A^4 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then which of the following options is the value of a+b+c+d? (1) 158 (2) 162 (3) 166 (4) 170 (5) 174

- 10. Each term in the five-term sequence real numbers  $a_1, a_2, a_3, a_4, a_5$  is greater than 1, and for any two adjacent terms, one is twice than the other. If  $a_1 = log_{10}36$ , how many possible values can  $a_5$  take?
  - (1) 3 (2) 4 (3) 5 (4) 7 (5) 8
- 11. As shown in the figure,  $\triangle ABC$  is an acute-angled triangle. *P* is a point outside the circumcircle  $\Gamma$  of  $\triangle ABC$ .  $\overline{PB}$  and  $\overline{PC}$  are both tangent to the circle. Assume  $\angle BPC = \theta$ , which of the following options is the value of cosA?
  - (1)  $\sin 2\theta$  (2)  $\frac{\sin \theta}{2}$  (3)  $\sin \frac{\theta}{2}$  (4)  $\frac{\cos \theta}{2}$  (5)  $\cos \frac{\theta}{2}$

12. Let f(x) be a cubic polynomial function with real coefficients. When (x+1)f(x) is divided by  $x^3 + 2$ , the remainder is x+2. If f(0) = 4, then which of the following options represents the value of f(2)? (1) 8 (2) 10 (3) 15 (4) 18 (5) 20

- 13. Peter makes a circular fence with radius 9 m. Anna, who does not want to be closed to Peter, goes 5 m away from him and builds a fence with radius 4 m. What is the minimum distance between de fences?
  - (1) 0
  - (2) 1
  - (3) 10
  - (4) 8
  - (5) 5

14. Let f be a polynomial such that  $f(x^2 + 1) = x^4 + 4x^2$ . Determine  $f(x^2 - 1)$ .

- (1)  $x^4 4x^2$
- (2)  $x^4$
- (3)  $x^4 + 4x^2 4$
- (4)  $x^4 4$
- (5) None of the above.



- 15. On the coordinate plane, the vertices of  $\triangle ABC$  are located at A(0, 2), B(1,0) and C(4,1). Please select the correct options:
  - (1) Among the sides of  $\triangle ABC$ ,  $\overline{BC}$  is the longest.
  - (2) sinA < sinC
  - (3)  $\triangle ABC$  is an acute triangle.

(4) 
$$sinB = \frac{7\sqrt{2}}{10}$$

- (5) The radius of the circumcircle of  $\triangle ABC$  is less than 2.
- 16. If you enter a positive integer N into a calculator, and then press the "√" key (to take the positive square root) three times consecutively, and the displayed result is 2, then N is equal to which of the following options?
  (1) 2<sup>3</sup> (2) 2<sup>4</sup> (3) 2<sup>6</sup> (4) 2<sup>8</sup> (5) 2<sup>12</sup>
- 17. Consider a circle on the coordinate plane with center at the origin O and a radius of 1, intersecting the coordinate axes at points A and B. Take a point C on the arc within the first quadrant and draw the tangent lines of the circle to intersect the two axes at points D and E, respectively, as shown in the figure. Let  $\angle OEC = \theta$ , please choose the correct option which represents  $tan\theta$ . (1)  $\overline{OE}$  (2)  $\overline{OC}$  (3)  $\overline{OD}$  (4)  $\overline{CE}$  (5)  $\overline{CD}$



18. A student deduced that two physical quantities *s*, *t* should satisfy an equation. To verify his theory, he conducted experiments and obtained 15 pieces of data  $(s_k,t_k)$  on two physical quantities.  $k = 1, \dots, 15$ . The teacher suggested that he takes the logarithm of  $t_k$  first and mark the corresponding points  $(s_k, \log t_k)$ ,  $k = 1, \dots, 15$  on the coordinate plane, as shown in the figure; the first data is the horizontal axis coordinate, and the second data is the vertical axis coordinate. Using regression line analysis, a certain student confirmed his theory. Which of the following options is most likely to be the relational expression of *s*, *t* obtained by this student?



(1) s = 2t (2) s = 3t (3)  $t = 10^{s}$  (4)  $t^{2} = 10^{s}$  (5)  $t^{3} = 10^{s}$ 

- 19. Research shows that after taking a certain drug, the amount of drug residue in the user's body decreases exponentially over time. It is known that 2 hours after administration, half of the initial dose remains in the body. Which of the following statements is correct?
  - (1)  $\frac{1}{3}$  of the drug remains in the body 3 hours after administration.
  - (2)  $\frac{1}{4}$  of the drug remains in the body 4 hours after administration.
  - (3)  $\frac{1}{6}$  of the drug remains in the body 6 hours after administration.
  - (4)  $\frac{1}{8}$  of the drug remains in the body 8 hours after administration.
  - (5)  $\frac{1}{10}$  of the drug remains in the body 10 hours after administration.
- 20. As shown in the figure, OABC DEFG is a cube. Which of the following vectors is parallel to the cross product  $\overrightarrow{AD} \times \overrightarrow{AG}$ ?

С

 $(1) \overline{AE}$ 



- $(4) \overrightarrow{DE} \qquad \qquad O \qquad \qquad A \qquad B$
- $(5)\overrightarrow{OE}$
- 21. Assume a∈ {-6,-4,-2,2,4,6} and a is the coefficient of the highest degree term of a cubic polynomial f(x) with real coefficients. If the graph of function y = f(x) intersects with the x-axis at three points, and their x coordinates form an arithmetic sequence with the first term -7 and common difference a. How many of a satisfy f(0) > 0?
  (1) 1 (2) 2 (3) 3 (4) 4 (5) 5
- 22. In the coordinate space, let O be the origin, and E be the plane defined by x z = 4. If the projection of the origin O onto the plane E is point Q, and the angle  $\alpha$  between the vector  $\overrightarrow{OQ}$  and the vector (1, 0, 0) is given, what is the value of  $\cos(\alpha)$  from the options below?

(1) 
$$-\frac{\sqrt{2}}{2}$$
 (2)  $-\frac{1}{2}$  (3)  $\frac{1}{2}$  (4)  $\frac{\sqrt{2}}{2}$  (5)  $\frac{\sqrt{3}}{2}$ 

23. The area of the square in the figure is *a* and the area of the circle is *b*. What is the area of the region delimited by the thick line?



- **(5)** *a* + *b*
- 24. A certain company has two new employees, A and B, who both started at the same time with the same starting salary. The company has promised the following salary adjustment methods for A and B:
  - A: After working for 3 months, starting from the next month, the monthly salary will increase by 200 dollars; thereafter, the salary will be adjusted in the same manner every 3 months.
  - B: After working for 12 months, starting from the next month, the monthly salary will increase by 1000 dollars; thereafter, the salary will be adjusted in the same manner every 12 months.

Based on the above description, please select the correct option.

- (1) After working for 8 months, the salary in the 9th month is 600 dollars higher than in the first month for employee A.
- (2) After working for one year, in the 13th month, employee A's salary is higher than employee B's.
- (3) After working for 18 months, in the 19th month, employee A's salary is lower than employee B's.
- (4) After working for 18 months, the total salary received by employee A is less than the total salary received by employee B.
- (5) After working for two years, in the 12 months of the third year, there are exactly 3 months where employee A's salary is higher than employee B's.
- 25. A newly opened beverage store launched three types of drinks: juice, milk tea, and coffee. The sales quantity (in cups) and total revenue (in dollars) for each type of drink over the first 3 days are shown in the table below. For example, on the first day, the sales volumes of juice, milk tea, and coffee were 60 cups, 80 cups, and 50 cups, respectively, with a total revenue of 1290 dollars. Assuming the price of each type of drink remains the same each day, what is the price per cup of coffee?

	Juice	milk tea	Coffee	Total revenue
	(cups)	(cups)	(cups)	(dollars)
Day 1	60	80	50	1290
Day 2	30	40	30	685
Day 3	50	70	40	1080

(1) 4 (2) 6 (3) 8 (4) 10 (5) 12

## **Answer Sheet**

1	1 □ 2 □ 3 ■ 4 □ 5 □	16	1 □ 2 □ 3 □ 4 ■ 5 □
2	1■ 2□ 3□ 4□ 5□	17	1 □ 2 □ 3 □ 4 □ 5 ■
3	1 □ 2 □ 3 ■ 4 □ 5 □	18	1 □ 2 □ 3 □ 4 ■ 5 □
4	1 □ 2 □ 3 □ 4 ■ 5 □	19	1 □ 2 ■ 3 □ 4 □ 5 □
5	1 □ 2 □ 3 ■ 4 □ 5 □	20	1 □ 2 □ 3 □ 4 □ 5 ■
6	1 ■ 2 □ 3 □ 4 □ 5 □	21	1 ■ 2 □ 3 □ 4 □ 5 □
7	1 □ 2 □ 3 □ 4 ■ 5 □	22	1 □ 2 □ 3 □ 4 ■ 5 □
8	1 □ 2 ■ 3 □ 4 □ 5 □	23	1 □ 2 ■ 3 □ 4 □ 5 □
9	1 □ 2 ■ 3 □ 4 □ 5 □	24	1 □ 2 □ 3 □ 4 □ 5 ■
10	1 □ 2 □ 3 ■ 4 □ 5 □	25	1 □ 2 □ 3 ■ 4 □ 5 □
11	1 □ 2 □ 3 ■ 4 □ 5 □		
12	1 □ 2 □ 3 □ 4 ■ 5 □		
13	1■ 2 □ 3 □ 4 □ 5 □		
14	1 □ 2 □ 3 □ 4 ■ 5 □		
15	1 □ 2 □ 3 □ 4 ■ 5 □		

## **Reference Formulas and Potentially Useful Values**

- 1. Pythagorean theorem:  $a^2 + b^2 = c^2$
- 2. Semicircle area:  $A = \frac{1}{2}\pi r^2$
- 3. Dot product:  $\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$ ; that is,  $\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$
- 4. Sphere volume:  $V_{sphere} = \frac{4}{3}\pi r^3$
- 5. Laws of logarithms:  $\log a \log b = \log \frac{a}{b}$ ;  $\log a + \log b = \log ab$
- 6. The discriminant of a quadratic equation  $ax^2 + bx + c = 0$  is given by  $\Delta = b^2 4ac$ , where  $\Delta$ (Delta) determines the nature of the roots:
  - If  $\Delta > 0$ , the equation has two distinct real roots;
  - If  $\Delta = 0$ , the equation has one real root (a repeated root);
  - If  $\Delta < 0$ , the equation has no real root,

While 
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 7. Matrix multiplication:  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ;  $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a \cdot a + b \cdot c & a \cdot b + b \cdot d \\ c \cdot a + d \cdot c & c \cdot b + d \cdot d \end{bmatrix}$
- 8. Reference number:  $\sqrt{2} \approx 0.414, \sqrt{3} \approx 1.732, \sqrt{5} \approx 2.236, \sqrt{6} = 2.449, \pi = 3.142$
- 9. Logarithmic value:  $log_{10}2 \approx 0.3010$ ,  $log_{10}3 \approx 0.4771$ ,  $log_{10}5 \approx 0.6990$ ,  $log_{10}7 \approx 0.8451$
- 10. Two angles are complementary if their sum is 90° ( $\theta_1 + \theta_2 = 90^\circ$ ), while  $\cos(90^\circ \theta) = \sin(\theta)$
- 11. Distance formula between two points  $(x_{1}, y_{1})$  and  $(x_{2}, y_{2})$ :  $d = \sqrt{(x_{2}, -x_{1})^{2} + (y_{2} y_{1})^{2}}$
- 12. To determine if a triangle contains an acute or obtuse angle:
  - if  $\cos \theta > 0, \theta$  is acute; if  $\cos \theta < 0, \theta$  is obtuse
- 13. Trigonometric calculation:
  - $\sin \theta = \frac{Opposite \ side}{Hypotenuse}$ ;  $\cos \theta = \frac{Adacent \ side}{Hypotenuse}$ ;  $\tan \theta = \frac{Opposite \ side}{Adacent \ side}$
  - Given that  $\sin^2 \theta + \cos^2 \theta = 1$ ; If  $0^\circ < \theta < 180^\circ$ ,  $\sin \theta = \sqrt{1 \cos^2 \theta}$
  - The sum formula for trigonometric functions:

sin(A + B) = sinAcosB + cosAsinBcos(A + B) = cosAcosB - sinAsinB $tan(A + B) = \frac{tanA + tanB}{1 - tanAtanB}$ 

- The law of Sines of  $\triangle ABC$ :  $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R$ , (*R* is the radius of the circumcircle of  $\triangle ABC$ )
- The cosine rule of  $\triangle ABC$ :  $c^2 = a^2 + b^2 2ab \cdot cosC$

14. The sum of the first n terms of an arithmetic sequence with the first term a and common difference d

is  $S = \frac{n(2a+(n-1)d)}{2}$ . The sum of the first *n* terms of a geometric sequence with the first term *a* and

common ratio r (where  $r \neq 1$ ) is  $S = \frac{a(1-r^n)}{1-r}$ .